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New solution of $D = 11$ supergravity on S^7 from $D = 4$

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Abstract: A new static partially twisted solution of $N = 4$, SO(4) gauged supergravity in $D = 11$ is obtained in this work using Cvetiĉ et al embedding of four dimensional into eleven dimensional supergravities. In four dimensions we get two solutions: an asymptotic one corresponding to AdS_4 and a near horizon fixed point solution of the form $AdS_2 \times H_2$. Hence, while the former solution has 32 supercharges the latter turns out to have only 4 conserved. Moreover, we managed to find an exact interpolating solution, thus connecting the above two. Aiming at a future study of AdS/CFT duality for the theory at hand we derived the Penrose limit of the four dimensional solutions. Interestingly the pp-wave limit of the near horizon solution suggests itself as being of the supernumerary supersymmetric type. In $D = 11$ we exhibit the uplift of the four dimensional solutions: one associated to $AdS_4 \times S^7$ and the other to a foliation of $AdS_2 \times H_2 \times S^7$, as well as their pp-wave limits.

Keywords: M-Theory, Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence, Supergravity Models.

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Contents

1. Introduction

Within the current form of string theory it is commonly accepted there exists a duality between superconformal field theories with large N and weakly coupled supergravities on AdS backgrounds [1]–[3]. Also non conformal theories have been recently studied [4]. Whenever there are solutions to supergravity theories preserving part of the original supersymmetry they might provide new evidence of the AdS/CFT duality establishing a correspondence with twisted SCYM theories. The twist term having origin in the projections imposed upon the Killing spinors [5, 6].

One way to look for AdS/CFT dualities is to obtain solutions from the supergravity side wrapping or twisting known BPS brane solutions of eleven or ten dimensional supergravities [7]. The solutions are determined using the first order supersymmetry equations to obtain the metric and field configurations and this tells us about the supersymmetry (partial or total) of the solution.

Alternatively one can uplift lower dimensional supergravity solutions to eleven or ten dimensions [8]–[10] which correspond, then, to a twisted or partially twisted supergravity in eleven or ten dimensions, respectively.

In this work we perform the uplifting procedure from $D = 4$ to $N = 4$, SO(4) gauged supergravity in $D = 11$ [8]: we use a metric ansatz for the four dimensional metric of the form $AdS_2 \times H_2$, where H_2 corresponds to a two dimensional hyperbolic space.

Starting from the uplift formulae in [8] we determined two solutions of the four dimensional supergravity. One is the vacuum AdS_4 with all matter fields set to zero which preserves the whole supersymmetry (32 supercharges). This admits an interpretation as describing an asymptotic behavior. The other solution has as matter contents two copies of SU(2) gauge fields pointing in one of the three S^3 directions. The two SU(2) gauge couplings being equal, and the scalar fields (dilaton and axion) set to zero. In this case there is the number of 4 supercharges conserved. This might be related to the equality of the coupling constants and the way the brane wraps onto the transverse S^7 [5, 6]. Here the interpretation corresponds to a near horizon regime since it includes the $r = 0$ region corresponding to the AdS_2 horizon. Interestingly we were able to find an exact interpolating solution connecting the above two.

As for the $AdS_2 \times H_2$ solution, when we approach the AdS_2 horizon, such a factor geometry dominates over the H_2. A low energy effective theory is obtained in $0 + 1$ dimensions which can be interpreted as a Super Conformal Quantum Mechanics that can be expressed in terms of $M2$ and $M5$ branes from the whole theory [11]–[13]. This is of interest as a way of studying (multi) black hole(s) configurations and the analysis of their quantum behavior .

Similar analysis to the one we present in this work have been made to find other supersymmetric solutions with diverse matter contents (see for example [14]–[16] and references therein).

In connection with AdS/CFT duality the parallel plane wave (Penrose) limits of supergravity scenarios have received much attention recently [17]–[21]. Such a limit could define a Matrix model to check the relation between the gravity and the gauge theory side [22] –which we do not pursue here. With this motivation we have obtained the Penrose limit of our solutions in $D = 4$ and determined that one of them exhibits a supernumerary supersymmetry [5, 6, 23]. It presents an enhancement of the original preserved supersymmetry. Such behavior has been observed in previous work [20].

This paper is organized as follows. In section 2 we provide the details in determining the two solutions in four dimensional supergravity. The uplifting to eleven dimensions is included. Section 3 is devoted to show the amount of supersymmetry preserved by the four dimensional solutions and we use this technic to obtain an exact interpolating solution between those obtained in the previous section. In section 4 the pp-wave limit of our solutions are obtained together with the amount of supersymmetry they preserve following an analogous analysis as in section 3. Finally section 5 contains a discussion of our results.

2. New solutions in $D = 4$ and $D = 11$

In this section we obtain a bosonic solution for the SO(4), $N = 4$ four dimensional supergravity described in [8] as follows. Since we would like to know what dual description such a solution admits in terms of a super conformal Yang-Mills theory we propose a background containing an AdS factor, so we begin with a metric ansatz of the form

$$ds_4^2 = f(r) ds^2_{AdS_2} + g(r) ds^2_{H_2} \, , \tag{2.1}$$

where H_2 is a two dimensional hyperbolic manifold and r is the radial coordinate of AdS_2.

A particular form of the SU(2) gauge fields with equal coupling constants and vanishing scalar fields are plugged in the resulting Einstein-Yang-Mills field equations. Since we want the solution to preserve some fraction of supersymmetry we look for field configurations satisfying the corresponding first order supersymmetry conditions [24]. We succeeded in doing so by introducing certain Killing spinor projections.

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We found two solutions. One corresponds to a near horizon configuration of the four dimensional theory, whereas the other can be read as representing an asymptotic behavior with all matter fields turned off.

Let us start with the four dimensional bosonic lagrangian for SO(4), $N = 4$ gauged supergravity [8]

$$\begin{aligned}\mathcal{L} = R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}e^{2\phi}(\partial_\mu\chi)^2 + 2\alpha^2(4 + 2\cosh\phi + \chi^2 e^{\phi}) - \frac{1}{2}e^{-\phi}F^2 - \\ - \frac{e^{\phi}}{2(1+\chi^2 e^{2\phi})}\tilde{F}^2 - \frac{\chi}{2}F * F + \frac{\chi e^{2\phi}}{2(1+\chi^2 e^{2\phi})}\tilde{F} * \tilde{F}\,.\end{aligned} \tag{2.2}$$

$A, \tilde{A}$ are SU(2) gauge fields with field strengths $F^i = dA^i + \frac{1}{2}\alpha\varepsilon^{ijk}A^jA^k$, and similarly for $\tilde{F}$. Here we have set the two coupling constants equal $g = \tilde{g} = \alpha$. ϕ, χ are scalar fields (dilaton and axion respectively).

From this we can write the Einstein equations

$$\begin{aligned}R_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{e^{2\phi}}{2}\partial_\mu\chi\partial_\nu\chi + e^{-\phi}\left(F_{\mu\alpha}F_\nu^\alpha - \frac{g_{\mu\nu}}{4}F^2\right) + \\ + \frac{e^{\phi}}{1+\chi^2 e^{2\phi}}\left(\tilde{F}_{\mu\alpha}\tilde{F}_\nu^\alpha - \frac{g_{\mu\nu}}{4}\tilde{F}^2\right) - 6\alpha^2 g_{\mu\nu}\,.\end{aligned} \tag{2.3}$$

Now we asume $A_\mu = \tilde{A}_\mu$. As for the YM equations of motion we get

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}e^{-\phi}F^{\mu\nu a}\right) = \left(A_\mu^b F^{\mu\nu c} - \frac{\chi}{\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}A_\mu^b F_{\alpha\beta}^c\right)\varepsilon^{abc}\,. \tag{2.4}$$

Our ansatz consists of setting $\phi = \chi = 0$, together with the metric (2.1):

$$ds^2 = e^{2f(r)}\left(-dt^2 + dr^2\right) + \frac{e^{2h(r)}}{y^2}\left(dx^2 + dy^2\right), \tag{2.5}$$

where $f(r), h(r)$ are functions to be determined.

A direct calculation using the YM equations (2.4) yields the following field solution

$$A_x^3 = \tilde{A}_x^3 = \frac{k}{y} \Rightarrow F_{xy}^3 = \tilde{F}_{xy}^3 = \frac{k}{y^2}\,; \qquad \phi = \chi = 0\,, \tag{2.6}$$

with k a constant to be determined later. Next we write the components of the Einstein equations for this configuration

$$\begin{aligned}R_{11} &= f'' + 2f'h'\,,\\ R_{22} &= -f'' - 2(h')^2 - 2h'' + 2f'h'\,,\\ R_{33} &= R_{44} = -\frac{1}{y^2}\left(2e^{2(h-f)}(h')^2 + e^{2(h-f)}h'' + 1\right).\end{aligned} \tag{2.7}$$

Now we let $h(r) = \text{const}$ (fixed point) and find

$$e^{-2h} = \frac{-1 + \sqrt{1 + 24(k\alpha)^2}}{2k^2}\,, \tag{2.8}$$

$$e^{2f(r)} = \frac{2}{F^2 + 12\alpha^2}\frac{1}{r^2}\,, \tag{2.9}$$

where $F^2 = 2k^2 e^{-4h}$. In light of (2.12) we obtain the metric corresponding to $AdS_2 \times H_2$. Due to the fact that the AdS_2-horizon lies on $r = 0$, the near horizon region is included within this description.

It is worth noticing that the matter free solution $A = \tilde{A} = 0$ to the above equations reads

$$ds^2 = \frac{2}{\alpha^2 r^2}(-dt^2 + dr^2 + dx^2 + dy^2)\,, \tag{2.10}$$

just AdS_4. Let us observe that (2.6) yields zero for $y \to \infty$ and since (2.10) corresponds to a matter free configuration, we refer to it as asymptotic.

Now we proceed to the uplifting from four to eleven dimensions [8]. We get

1. $AdS_4 \times S^7$:

$$ds^2_{11} = \frac{2}{\alpha^2 r^2}(-dt^2 + dr^2 + dx^2 + dy^2) + \frac{2}{\alpha^2}d\xi^2 + \frac{1}{2\alpha^2}\left[\cos^2\xi d\Omega_3 + \sin^2\xi d\tilde{\Omega}_3\right], \tag{2.11}$$

where $d\Omega_3$ is the metric on the three sphere.

2. $AdS_2 \times H_2 \times \tilde{S}^7$, tilde meaning the squashing of S^7

$$\begin{aligned} ds^2_{11} &= \frac{2}{F^2 + 12\alpha^2}\frac{1}{r^2}\left(-dt^2 + dr^2\right) + \frac{e^{2h}}{y^2}\left(dx^2 + dy^2\right) + \frac{2}{\alpha^2}d\xi^2 + \\ &+ \frac{1}{2\alpha^2}\left[\cos^2\xi\left(\sigma_1^2 + \sigma_2^2 + \left[\sigma_3 - \frac{dx}{y}\right]^2\right) + \sin^2\xi\,(\sigma \leftrightarrow \tilde{\sigma})\right]. \end{aligned} \tag{2.12}$$

In this last equation σ^i are left invariant one forms[1] in SO(3) $\simeq$ SU(2).

For completeness we write down the gauge and strength fields. In four dimensions we have

$$\begin{aligned} A^i = \tilde{A}^i &= \frac{k}{y}dx\ \delta^i_3\,, \\ F^i = \tilde{F}^i &= \frac{k}{y^2}dx \wedge dy\ \delta^i_3\,. \end{aligned} \tag{2.14}$$

In the case of eleven dimensions we adopt the one forms of [8]

$$h^i = \sigma^i - \alpha A^i\,, \tag{2.15}$$

where A^i is given in (2.14), and similarly for $\tilde{h}^i$. Then the eleven dimensional four form field strength reads

$$\begin{aligned} F_{(4)} &= -3\sqrt{2}\alpha\varepsilon_{(4)} + F'_{(4)}\,, \\ \sqrt{2}\alpha^2 F'_{(4)} &= \sin\xi^2\cos\xi^2 d\xi \wedge \left(h^i \wedge F^i - \bar{h}^i \wedge \bar{F}^i\right) + \\ &+ \frac{1}{4}\epsilon_{ijk}\left(\cos\xi^2 h^i \wedge h^j \wedge *F^k + \sin\xi^2 \bar{h}^i \wedge \bar{h}^j \wedge *\bar{F}^k\right). \end{aligned} \tag{2.16}$$

[1]

$$\begin{aligned} \sigma^1 &= d\rho + \cos\zeta d\tau\,, \\ \sigma^2 &= \cos\rho d\zeta + \sin\zeta\sin\rho d\tau\,, \\ \sigma^3 &= \sin\rho d\zeta - \sin\zeta\cos\rho d\tau\,. \end{aligned} \tag{2.13}$$

The near horizon and asymptotic character of our solutions suggests to look for a metric that connects smoothly between the two. Since we will use the supersymmetry transformation equations we postpone this analysis until the next section. Also we want to determine what is the fraction of supersymmetry retained by this solution. We pursue this in the next section.

3. Supersymmetry of the solutions

As for our AdS_4 solution it is well known it is maximally supersymmetric [25]–[29], namely it has 32 supercharges.

Regarding the near horizon solution (2.12) we now determine the amount of supersymmetry preserved. To this end we use the supersymmetry transformations [24] and [30]–[32]

$$\delta\bar{\chi}^i = \frac{1}{2\sqrt{2}}\varepsilon^{ijkl}\bar{\epsilon}^j\sigma^{\mu\nu}F^{kl}_{\mu\nu} = 0\,, \tag{3.1}$$

$$\delta\bar{\psi}^i_\lambda = \bar{\epsilon}^i\overleftarrow{D}_\lambda - \frac{1}{2}\bar{\epsilon}^j\gamma_\lambda\sigma^{\mu\nu}F^{ij}_{\mu\nu} - \alpha\bar{\epsilon}^i\gamma_\lambda = 0\,, \tag{3.2}$$

where $D_\mu\psi^I_\nu = (\partial_\mu + \frac{1}{2}\omega_{\mu ab}\sigma^{ab})\psi^i_\nu + 2\alpha A^{ij}_\mu\psi^j_\nu$, and $\omega_{a\mu\nu}$ is the spin connection. Our conventions are

$$\sigma^{ab} = \frac{1}{4}[\gamma^a,\gamma^b]\,, \qquad \gamma^\mu = e^\mu_a\gamma^a\,, \tag{3.3}$$

and

$$\gamma^0 = i\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0\end{pmatrix}, \qquad \gamma^1 = -\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3\end{pmatrix},$$
$$\gamma^2 = i\begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0\end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1\end{pmatrix}. \tag{3.4}$$

Here latin indices are flat index and greek indices are curved.

We also need the spin connection for (2.5). We get

$$\omega_{t\hat{t}\hat{r}} = \frac{1}{r}\,,$$
$$\omega_{x\hat{y}\hat{x}} = \frac{1}{y}\,. \tag{3.5}$$

Combining (2.14) and (3.1) we have $\varepsilon^1 = \varepsilon^4 = 0$.

Now we write eq. (3.2) in components

$$\bar{\varepsilon}^2\left(\overleftarrow{\partial}_t + \frac{1}{r}\sigma^{\mathrm{tr}} + \alpha e^f\gamma^t\right) + \bar{\varepsilon}^3\left(ke^{f-2h}\gamma^t\sigma^{xy}\right) = 0\,, \tag{3.6}$$

$$\bar{\varepsilon}^2\left(\overleftarrow{\partial}_r - \alpha e^f\gamma^r\right) - \bar{\varepsilon}^3\left(ke^{f-2h}\gamma^r\sigma^{xy}\right) = 0\,, \tag{3.7}$$

$$\bar{\varepsilon}^2\left(\overleftarrow{\partial}_x + \frac{1}{y}\sigma^{yx} - \frac{\alpha e^h}{y}\gamma^x\right) + \bar{\varepsilon}^3\left(\frac{2\alpha k}{y} - \frac{k}{e^h y}\gamma^x\sigma^{xy}\right) = 0\,, \tag{3.8}$$

$$\bar{\varepsilon}^2\left(\overleftarrow{\partial}_y - \frac{\alpha e^h}{y}\gamma^y\right) - \bar{\varepsilon}^3\left(\frac{k}{e^h y}\gamma^y\sigma^{xy}\right) = 0\,, \tag{3.9}$$

Imposing $\partial_\mu \bar{\varepsilon}^2 = \partial_\mu \bar{\varepsilon}^3 = 0$, $\mu = t, x, y$ on the spinors as in [33], the projections, with $i = 2, 3$,

$$\begin{aligned} \bar{\varepsilon}^i \gamma^x \gamma^y &= i \bar{\varepsilon}^i \,, \\ \bar{\varepsilon}^i \gamma^r &= \bar{\varepsilon}^i \,, \end{aligned} \tag{3.10}$$

and substituting everything in (3.6) we finally end up with

$$\begin{aligned} \bar{\varepsilon}^2 &= i \bar{\varepsilon}^3 \,, \\ \bar{\varepsilon}^2(r) &= \sqrt{r} \bar{\varepsilon}_0 \,, \end{aligned} \tag{3.11}$$

which solve (3.1), (3.2) with metric (2.5) and (2.8), (2.9).

The projection (3.10) together with (3.11) reduce the number of independent spinor components of ϵ^2, ϵ^3 to 4. Using the triviality of ϵ^1 and ϵ^4 we conclude that the fraction of supersymmetry preserved by the near horizon solution is $1/8$.

This corresponds to 4 supercharges retained by this supergravity solution in four dimensions. Remarkably such a number of supercharges is not commonly found. Let us stress this number stems from the twisting given by the projection (3.10).

Given our near horizon solution $AdS_2 \times H_2$ in four dimensions it is interesting to see what happens in the IR: as we lower the energy of the massless modes their wave length becomes large compared to the size of the (x, y) coordinates in the transverse space and there is no dependence in those coordinates; so we end up with an effectively $0 + 1$ dimensional field theory. So, the supergravity theory at hand is dual to a $2 + 1$ super conformal Yang-Mills theory and at low energies is dual to a Super Conformal Quantum Mechanics with four supercharges (see [11]–[13]) whose field description may be given in terms of the M2brane of the full theory.

Next we compute the interpolating solution we mentioned at the end of the previous section. To this end we use again the ansatz (2.1)

$$ds^2 = e^{2f(r)} \left(-dt^2 + dr^2\right) + \frac{e^{2h(r)}}{y^2} \left(dx^2 + dy^2\right) . \tag{3.12}$$

As the Einstein equations are second order and non linear, and we want the solution to preserve some supersymmetry we impose the supersymmetry first order condition to the above ansatz. We use the formulae given above.

The non zero spin connection components for this metric are

$$\begin{aligned} \omega_{t01} &= f' \,, \\ \omega_{x12} &= \omega_{y13} = e^{h-f} \frac{h'}{y} \,, \\ \omega_{x23} &= \frac{1}{y} \,, \end{aligned} \tag{3.13}$$

where the prime denotes derivative with respect to r. From (3.2) the supersymmetry condition now reads[2]

$$\begin{aligned}
0 &= \bar{\epsilon}^i \left(f'\sigma^{01} + \alpha e^f \gamma^0\right) + \bar{\epsilon}^j \left(-\sigma^{23}\gamma^0 k e^{-2h+f}\right) M^{ij}\,, \\
0 &= \bar{\epsilon}^i \left(e^{h-f} h'\sigma^{12} + \sigma^{23} - \alpha e^h \gamma^2\right) + \bar{\epsilon}^j \left(2\alpha k + \sigma^{23}\gamma^2 k e^{-h}\right) M^{ij}\,, \\
0 &= \bar{\epsilon}^i \left(e^{h-f} h'\sigma^{13} - \alpha e^h \gamma^3\right) + \bar{\epsilon}^j \left(\sigma^{23}\gamma^3 k e^{-h}\right) M^{ij}\,,
\end{aligned} \tag{3.14}$$

plus an equation yielding the radial dependence of the spinor.

We can use the projection $\bar{\epsilon}^j \sigma^{23} M^{ij} = \bar{\epsilon}^i$. Substituting back in the last system we obtain from the first and third equations $h'(r) = f'(r)$. If we use $h(r) = f(r)$ and the second equation we obtain that $\alpha \propto k$ and

$$f(r) = h(r) = \ln \frac{\tanh\left(r + C_1\right)}{C_2 \alpha}\,. \tag{3.15}$$

We have obtained a near horizon and an asymptotic solution which is interpolated by (3.15) in four dimensions, that corresponds to a M2 brane solution to M theory.

4. Penrose limit in $D = 4$ and $D = 11$

In this section we obtain the Penrose limit of our four dimensional solutions (2.5), (2.8) (2.9), (2.10) that corresponds to pp-wave configurations. We include also the uplifting from four to eleven dimensions. In the sequel we follow [20] and [34]–[36].

Let us start with the near horizon solution and write it down as

$$ds^2 = e^{2f(r)}\left(-dt^2 + dr^2\right) + e^{2h}\left(d\theta^2 + \sinh\theta^2 d\phi^2\right). \tag{4.1}$$

Since we want to make the limit on a null geodesic on $\theta = \sinh^{-1} 1, \quad \phi = 0$ we introduce new coordinates (u, v, p, q) by

$$\begin{aligned}
t &= \frac{1}{2}\Omega\left(u - \Omega^2 v\right), \\
r &= \frac{1}{2}\Omega\left(u + \Omega^2 v\right), \\
\theta &= \Omega p + \sinh^{-1} 1\,, \\
\phi &= \Omega q\,.
\end{aligned} \tag{4.2}$$

These coordinates are motivated by the coupling-dependent factor in (2.5), (2.8), (2.9). We need to redefine the gauge fields so that all the terms in the lagrangian (2.2) acquire the same factor of Ω. This can be done redefinition of the following fields

$$\begin{aligned}
\bar{g}_{\mu\nu} &= \Omega^{-2} g_{\mu\nu}\,, \\
\bar{A} &= \Omega^{-1} A\,, \\
\bar{\alpha} &= \Omega\alpha\,.
\end{aligned} \tag{4.3}$$

[2] We use $M^{ij} = \delta_2^{[i}\delta_3^{j]}$.

After performing the limit $\Omega \to 0$ the metric and the fields read

$$d\bar{s}^2 = 2k^2 \left[\frac{1}{u^2} (dudv) + \left(dp^2 + dq^2 \right) \right] , \tag{4.4}$$

$$\bar{A}^3 = \frac{k}{\sqrt{2}} (dp + dq) , \tag{4.5}$$

$$\bar{F}^3 = 0 . \tag{4.6}$$

In order to make the uplifting to 11 dimensions of this solution we need to make a similar rescaling on the remaining seven coordinates of the transverse space as follows

$$\xi \to \Omega^2 \xi + \frac{\pi}{4} , \tag{4.7}$$

and homogeneous transformation for the angular coordinates (ρ, ζ, τ) parameterizing the left invariant one-forms. Altogether we obtain

$$\begin{aligned} d\bar{s}^2 = & \ 2k^2 \left[\frac{1}{u^2} (dudv) + 2 \left(dp^2 + dq^2 \right) \right] + \\ & \frac{1}{4\alpha^2} \left[2d\xi^2 + \left(d\rho^2 + d\zeta^2 + d\tau^2 + 2d\zeta d\tau \right) + \left(d\tilde{\rho}^2 + d\tilde{\zeta}^2 + d\tilde{\tau}^2 + 2d\tilde{\zeta} d\tilde{\tau} \right) \right] . \end{aligned} \tag{4.8}$$

Next we take the Penrose limit of the asymptotic solution (2.11). To this end we propose the following change of coordinates and metric scaling

$$\begin{aligned} t &= \left(u - \Omega^2 v \right) , \\ r &= \left(u + \Omega^2 v \right) , \\ \bar{g}_{\mu\nu} &= \Omega^{-2} g_{\mu\nu} . \end{aligned} \tag{4.9}$$

Because the asymptotic solution does not have any coupling dependence the rescaling and redefinition are the usual ones [34].

Thus we obtain

$$d\bar{s}^2 = \frac{8}{\alpha^2 u^2} \left(dudv + dx^2 + dy^2 \right) . \tag{4.10}$$

Regarding the uplifting to eleven dimensions we get

$$\begin{aligned} d\bar{s}^2 = & \ \frac{1}{\alpha^2} \left[\frac{8}{u^2} \left(dudv + dx^2 + dy^2 \right) + 2d\xi^2 + \right. \\ & \left. + \left(d\tilde{\rho}^2 + d\tilde{\zeta}^2 + d\tilde{\tau}^2 + 2d\tilde{\zeta} d\tilde{\tau} \right) + \left(d\tilde{\rho}^2 + d\tilde{\zeta}^2 + d\tilde{\tau}^2 + 2d\tilde{\zeta} d\tilde{\tau} \right) \right] . \end{aligned} \tag{4.11}$$

In this form we obtain pp-wave configurations as limits of our solutions with and without gauge fields. Next we determine the amount of supersymmetry preserved by these pp-wave configurations.

AdS_4**.** It is straightforward to see this limit preserves all the supersymmetries since the original AdS_4 solution does and what we do corresponds to a renaming of coordinates.

$AdS_2 \times H_2$. From (3.1) and (4.6) we see that the condition $\epsilon_1, \epsilon_4 = 0$ of the near horizon supersymmetry analysis no longer holds; furthermore the equation (3.2) now reads

$$\delta\bar{\psi}^i_\lambda = \bar{\epsilon}^i \overleftarrow{D}_\lambda - \alpha\bar{\epsilon}^i \gamma_\lambda = 0\,. \tag{4.12}$$

Hence

$$\bar{\epsilon}^i \left(\overleftarrow{\partial}_\mu + \frac{1}{2}\omega_{\mu ab}\sigma^{ab} - \alpha\gamma_\mu \right) + \bar{\epsilon}^j \left(2\alpha A^{ij}_\mu \right) = 0\,. \tag{4.13}$$

For $i = 1, 4$ we can verify that $\epsilon^1, \epsilon^4 = \text{const}$ is always a solution of (3.2). The non-zero spin connection for this configuration is

$$\omega_{u\hat{u}\hat{v}} = \frac{1}{2u}\,. \tag{4.14}$$

Plugging everything into (3.2) we obtain the following equations

$$\bar{\epsilon}^2 \left(\overleftarrow{\partial}_u + \frac{1}{2u}\sigma^{\hat{p}\hat{q}} - \alpha\gamma_u \right) = 0\,, \tag{4.15}$$

$$\bar{\epsilon}^2 \left(\overleftarrow{\partial}_v - \alpha\gamma_v \right) = 0\,, \tag{4.16}$$

$$\bar{\epsilon}^2 (\overleftarrow{\partial}_p - \alpha\gamma_p) + \sqrt{2}\alpha k \bar{\epsilon}^3 = 0\,, \tag{4.17}$$

$$\bar{\epsilon}^2 (\overleftarrow{\partial}_q - \alpha\gamma_q) + \sqrt{2}\alpha k \bar{\epsilon}^3 = 0\,, \tag{4.18}$$

and similarly for $\epsilon^2 \leftrightarrow \epsilon^3$. Let us observe that $\epsilon^i = \epsilon^i(u)$ solves (4.15)–(4.18), and then we find that $\epsilon^2 \propto \epsilon^3$.

So we have that the pp-wave limit of the $AdS_2 \times H_2$ enhances the supersymmetry to 3/4 of the total, which means there are 24 supercharges conserved. As described in [5, 6, 23] this corresponds to a supernumerary supersymmetry.

5. Discussion

In this work we have obtained two supersymmetric solutions to $D = 4$, $N = 4$, SO(4) gauged supergravity, given by (2.5), (2.8), (2.9) and (2.10), which upon uplifting become solutions of $D = 11$ supergravity on S^7. Our analysis was based on Cvetiĉ et al embedding [8] between these two theories. While one of the solutions is the known AdS_4 vacuum, the other one corresponds to $AdS_2 \times H_2$. We refer to the former as asymptotic because it comes from setting every matter field to zero which corresponds to their spatial asymptotic behavior. As for the latter we call it near horizon since near the AdS_2 horizon its contribution dominates over the H_2. The supersymmetry preserved by the solutions is, respectively, full (asymptotic) and 1/8 (near horizon) of the original supersymmetry in four dimensions. We were able to find an exact interpolating solution between the two.

In the case of $AdS_2 \times H_2$ the IR flow can be obtained when the wave length of the massless modes becomes large at low energies compared with the size of the transverse space, thus dropping all dependence in those coordinates. Thus, the supergravity theory

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here studied is dual to a $2+1$ dimensional Yang-Mills theory which in the low energy limit becomes effectively 0+1 dimensional corresponding to a superconformal Quantum Mechanics with 4 supercharges, a number not commonly found.

Looking forward to study AdS/CFT dualities of the kind of theories here considered we derived the Penrose limit of our $AdS_2 \times H_2$ solution as given by (4.4). The supersymmetry this limit preserves is 3/4 of the whole which is 24 supercharges, corresponding to a supernumerary type [5, 6, 23]. Uplifting the $D=4$ pp-wave limit of $AdS_2 \times H_2$ (4.4) to eleven dimensions gave us the $D=11$ pp-wave form (4.8).

For completeness we have included the Penrose limit of the AdS_4 solution (4.10) which is maximally supersymmetric. Its uplifting to $D=11$ was performed giving (4.11).

It would be of interest to check whether our analysis can be extended in a more general context, for example, containing families of supergravity solutions (see for instance [37]), at least as general as the Cvetiĉ et al analysis allows to do. Furthermore, either in our case or a more general one looking for the dual SYM theory might shed light on the AdS/CFT correspondence. We leave such analysis for future work.

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